السكشن الثالث 12/11/2020

الفرقة الثانية بيولوجى + جيولوجيا

التفاضل المتقدم

Ex: Discuss the existence of the limits:

$$(1)\ \lim_{(x,y)\to(0,0)} \frac{5xy}{x^2+3y^2}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0} \frac{5x(mx)}{x^2+3(mx)^2} = \lim_{x\to 0} \frac{5mx^2}{x^2+3m^2x^2} = \lim_{x\to 0} \frac{5mx^2}{x^2(1+3m^2)} = \boxed{\frac{5m}{(1+3m^2)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(2)\ \lim_{(x,y)\to(0,0)} \frac{5xy^3}{x^4+y^4}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0} \frac{5x(mx)^3}{x^4+(mx)^4} = \lim_{x\to 0} \frac{5mx^4}{x^4+m^4x^4} = \lim_{x\to 0} \frac{5mx^4}{x^4(1+m^4)} = \boxed{\frac{5m}{(1+m^4)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(3)\ \lim_{(x,y)\to(0,0)} \sqrt{\frac{x^3}{x^3+y^3}}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0} \sqrt{\frac{x^3}{x^3+(mx)^3}} = \lim_{x\to 0} \sqrt{\frac{x^3}{x^3+m^3x^3}} = \lim_{x\to 0} \sqrt{\frac{\cancel{x^3}}{\cancel{x^3}(1+m^3)}} = \boxed{\sqrt{\frac{1}{(1+m^3)}}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(4)\ \lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^3}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0} \frac{x^2(mx)}{x^3+(mx)^3} = \lim_{x\to 0} \frac{mx^3}{x^3+m^3x^3} = \lim_{x\to 0} \frac{m\cancel{x^3}}{\cancel{x^3}(1+m^3)} = \boxed{\frac{m}{(1+m^3)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

على محمد يوسف الهلالى

المعيد بقسم الرياضيات

$$(5)\ \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+3y^3}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0}\frac{x(mx)^2}{x^3+3(mx)^3}=\lim_{x\to 0}\frac{x(m^2x^2)}{x^3+3(m^3x^3)}=\lim_{x\to 0}\frac{x^3 m^2}{x^3(1+3m^3)}=\boxed{\frac{m^2}{(1+3m^3)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(6)\ \lim_{(x,y)\to(0,0)} \frac{x+y}{x-y}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0}\frac{x+(mx)}{x-(mx)}=\lim_{x\to 0}\frac{x(1+m)}{x(1-m)}=\boxed{\frac{(1+m)}{(1-m)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(7)\ \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)^2}{x^4+y^4}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0}\frac{\left(x^2+(mx)^2\right)^2}{x^4+(mx)^4}=\lim_{x\to 0}\frac{(x^2+m^2x^2)^2}{x^4+m^4x^4}=\lim_{x\to 0}\frac{x^4(1+m^2)^2}{x^4(1+m^4)}=\boxed{\frac{(1+m^2)^2}{(1+m^4)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(8)\ \lim_{(x,y)\to(0,0)} \frac{x+y^2}{x^2+y}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0}\frac{x+(mx)^2}{x^2+(mx)}=\lim_{x\to 0}\frac{x+m^2x^2}{x^2+mx}=\lim_{x\to 0}\frac{x(1+m^2x)}{x(x+m)}=\lim_{x\to 0}\frac{(1+m^2x)}{(x+m)}=\frac{(1+0)}{(0+m)}=\boxed{\frac{1}{m}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(9)\ \lim_{(x,y)\to(0,0)} \frac{x^2-3xy+2y^2}{4x^2+5y^2}$$

Taking the general path $y = mx$, we have

$$\lim_{x\to 0}\frac{x^2-3x(mx)+2(mx)^2}{4x^2+5(mx)^2}=\lim_{x\to 0}\frac{x^2-3mx^2+2m^2x^2}{4x^2+5m^2x^2}=\lim_{x\to 0}\frac{x^2(1-3m+2m^2)}{x^2(4+5m^2)}=\boxed{\frac{(1-3m+2m^2)}{(4+5m^2)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(10)\ \lim_{(x,y)\to(1,0)} \frac{xy-y}{x^2-2x+y^2+1}$$

Taking the general path $y=mx$, we have

$$\lim_{(x,y)\to(1,0)} \frac{y(x-1)}{(x-1)^2+y^2} = \lim_{x\to1} \frac{(m(x-1))(x-1)}{(x-1)^2+(m(x-1))^2} = \lim_{x\to1} \frac{m\cancel{(x-1)^2}}{\cancel{(x-1)^2}(1+m^2)} = \boxed{\frac{m}{(1+m^2)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(11)\ \lim_{(x,y)\to(0,0)} \frac{x^2-3y}{2x^2+6y}$$

Taking the general path $y=mx$, we have

$$\lim_{x\to0} \frac{x^2-3(mx)}{2x^2+6(mx)} = \lim_{x\to0} \frac{x^2-3mx}{2x^2+6mx} = \lim_{x\to0} \frac{x(x-3m)}{x(2x+6m)} = \lim_{x\to0} \frac{(x-3m)}{(2x+6m)} = \frac{-3m}{6m} = \boxed{\frac{-1}{2}}$$

نلاحظ أن النهاية لا تعتمد على المسار m ولكن هذا لا يكفى أن نقول ان النهاية موجودة لذا يمكن ايجاد النهاية باستخدام مسار عام اخر وليكن $y=mx^2$

$$\lim_{x\to0} \frac{x^2-3(mx^2)}{2x^2+6(mx^2)} = \lim_{x\to0} \frac{x^2-3mx^2}{2x^2+6mx^2} = \lim_{x\to0} \frac{x^2(1-3m)}{x^2(2+6m)} = \boxed{\frac{(1-3m)}{(2+6m)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

$$(12)\ \lim_{(x,y)\to(1,2)} \frac{xy-2x-y+2}{x^2+y^2-2x-4y+5}$$

Firstly: we note that we can find the path through the point $(1,2)$ as follows:

$$\frac{y-y_0}{x-x_0} = m \Rightarrow \frac{y-2}{x-1} = m \Rightarrow \boxed{(y-2)=m(x-1)}$$

And By simplifying the limit we have

$$\lim_{(x,y)\to(1,2)} \frac{x(y-2)-(y-2)}{x^2-2x+1+y^2-4y+4} = \lim_{(x,y)\to(1,2)} \frac{(y-2)(x-1)}{(x-1)^2+(y-2)^2}$$

Taking the general path $(y-2)=m(x-1)$, we have

$$\lim_{x\to1} \frac{m(x-1)^2}{(x-1)^2+m^2(x-1)^2} = \lim_{x\to1} \frac{m(x-1)^2}{(x-1)^2(1+m^2)} = \boxed{\frac{m}{(1+m^2)}}$$

$\because$ the value of the limit depends on the general path, therefore, the limit **doesn't exist.**

Ex: Find the value of the limits:

$(1)\ \lim_{(x,y)\to(1,2)} 2x+3y$ $\qquad \lim_{(x,y)\to(1,2)} 2x+3y = 2(1)+3(2) = \boxed{8}$

$(2)\ \lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y}$

بالتعويض المباشر نجد ان قيمة النهاية تساوى $\frac{0}{0}$ لذا نلجأ للتحليل

$$\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(1,1)} \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}} = \lim_{(x,y)\to(1,1)} (x+y) = 1+1 = \boxed{2}$$

$(3)\ \lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x+y}$ $\qquad \lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x+y} = \frac{1-1}{2} = \boxed{0}$

$(4)\ \lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2-y^2}$

بالتعويض المباشر نجد ان قيمة النهاية تساوى $\frac{0}{0}$ لذا نلجأ للتحليل

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2-y^2} = \lim_{(x,y)\to(0,0)} \frac{\cancel{(x^2-y^2)}(x^2+y^2)}{\cancel{(x^2-y^2)}} = \lim_{(x,y)\to(0,0)} (x^2+y^2) = 0+0+\boxed{0}$$

$(5)\ \lim_{(x,y)\to(2,2)} \frac{2x^2-5xy+3y^2}{x^2+3xy-4y^2}$

بالتعويض المباشر نجد ان قيمة النهاية تساوى $\frac{0}{0}$ لذا نلجأ للتحليل

$$\lim_{(x,y)\to(2,2)} \frac{2x^2-5xy+3y^2}{x^2+3xy-4y^2} = \lim_{(x,y)\to(2,2)} \frac{\cancel{(x-y)}(2x-3y)}{\cancel{(x-y)}(x+4y)} = \lim_{(x,y)\to(2,2)} \frac{(2x-3y)}{(x+4y)} = \frac{4-6}{2+8} = \frac{-2}{10} = \boxed{\frac{-1}{5}}$$

تحياتى للجميع

علـــى الهلالى

12/11/2020