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Optical soliton perturbation with Kundu–Eckhaus equation by exp $(-\phi(\xi))$-expansion scheme and G'/G^2-expansion method

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ABSTRACT

This paper reveals dark and singular optical solitons, as well as their combinations thereof, of the perturbed Kundu–Eckhaus equation. Two integration methodologies are adopted here. The existence criteria for these solitons are also presented.

1. Introduction

Optical soliton perturbation is the backbone of telecommunications industry. This industry stays in business because of the marvel of soliton transmission technology. One of the various models that govern these pulse transmission across inter–continental distances is the Kundu–Eckhaus (KE) equation. In the past, several approaches were applied to address this model [1–10]. This paper studies the model with a few Hamiltonian perturbation terms by using two integration schemes. These are $\exp(-\phi(\xi))$-expansion scheme and G'/G^2-expansion method. The dark and singular soliton solutions will emerge from these integration schemes. These soliton solutions will be listed with their corresponding constraint conditions that will formulate the existence criteria of these pulses. The paper starts with an introduction to the model followed by a quick review of the two algorithms. Finally, the soliton solutions to the model will be computed. The details are discussed in the rest of the paper.

1.1. Governing equation

Consider the perturbed KE equation given as

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$$iu_\tau + au_{xx} + b|u|^4u = i[\alpha u_x + \lambda_1(|u|^2u)_x + \mu_1(|u|^2)_x u]. \tag{1}$$

In Eq. (1), x and τ represent spatial and temporal co–ordinates respectively. The dependent variable is $u(x, \tau)$ which gives the pulse profile. The first term in (1) is accounts for temporal evolution of the pulse, while the real-valued constants a, and b represents group velocity dispersion, quintic nonlinearity. On the right-hand side of (1), α denotes the inter-modal dispersion, λ_1 gives the effect of self–steepening for short pulses to eliminate the shock formation and μ_1 is the higher–order dispersion coefficient.

2. A quick glance at proposed analytical techniques

A general form of nonlinear evolution equation (NLEE) is:

$$P(u, D_\tau u, D_x u, D_\tau^2 u, D_{x\tau} u, D_x^2 u, \ldots) = 0, \tag{2}$$

where $u = u(x, \tau)$ is an unknown function, P is a polynomial in u as well as its partial derivatives. Here, nonlinear terms and its highest order derivatives are included. The traveling wave hypothesis is the transformation

$$u(x, \tau) = U(\xi), \quad \xi = x - v\tau.$$

After applying this wave transformation, the NLEE converts to a nonlinear ordinary differential equation (ODE) as given by:

$$S(U, U', U'', U''', \ldots) = 0, \tag{3}$$

where $'$ denotes the derivative with respect to ξ.

2.1. exp($-\phi(\xi)$)-expansion method

For exp($-\phi(\xi)$)-expansion method, the wave forms are expressed as

$$U(\xi) = \sum_{n=0}^{N} F_n(\exp(-\Phi(\xi)))^n, \tag{4}$$

where F_n are unknown constants to be determined and $\Phi(\xi)$ satisfies the auxiliary ODE:

$$\Phi'(\xi) = \exp(-\Phi(\xi)) + \mu\exp(\Phi(\xi)) + \lambda. \tag{5}$$

The auxiliary Eq. (5) has the general solutions given by one of the following five forms:

Case-1:(Hyperbolic function solutions)
When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, then

$$\Phi_1(\xi) = \ln\left[\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C)\right) - \lambda}{2\mu}\right]. \tag{6}$$

Case-2: (Trigonometric function solutions)
If $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, then

$$\Phi_2(\xi) = \ln\left[\frac{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + C)\right) - \lambda}{2\mu}\right]. \tag{7}$$

Case-3: (Hyperbolic function solutions)
However, if $\lambda^2 - 4\mu > 0$ and $\mu = 0$ and $\lambda \neq 0$ then

$$\Phi_3(\xi) = -\ln\left[\frac{\lambda}{\cosh(\lambda(\xi + C)) + \sinh(\lambda(\xi + C)) - 1}\right]. \tag{8}$$

Case-4: (Rational function solutions)
Next, if $\lambda^2 - 4\mu = 0$ and $\mu \neq 0$ and $\lambda \neq 0$,

$$\Phi_4(\xi) = \ln\left[-\frac{2(\lambda(\xi + C)) + 2}{\lambda^2(\xi + C)}\right]. \tag{9}$$

Case-5:
Finally, if $\lambda^2 - 4\mu = 0$ and $\mu = 0$ and $\lambda = 0$, then

$$\Phi_5(\xi) = \ln(\xi + C). \tag{10}$$

where C is the integration constant.

The value of N is obtained once we balance highest order nonlinear term with the highest order of derivative in U in Eq. (3). Substitution of Eq. (4) into Eq. (3) leads to algebraic equation involving powers of $\exp(-\Phi(\xi))$. Equating the coefficients of each power of $\exp(-\Phi(\xi))$ to zero yields a system of algebraic equations for F_n. Substituting F_n in Eq. (4), a variety of exact solutions of Eq. (2) can be exhibited.

2.2. G'/G^2-expansion method

For G'/G^2-expansion method, traveling wave solution is written as:

$$U(\xi) = a_0 + \sum_{n=1}^{N}\left\{\alpha_n\left(\frac{G'}{G^2}\right)^n + \beta_n\left(\frac{G'}{G^2}\right)^{-n}\right\}, \tag{11}$$

where $G = G(\zeta)$ satisfies

$$\left(\frac{G'}{G^2}\right)' = \mu + \lambda\left(\frac{G'}{G^2}\right)^2, \tag{12}$$

with $\lambda \neq 0$, $\mu \neq 1$ being integers. The unknown constants a_0, α_n, β_n $(n = 1, 2, 3, \ldots, N)$ must be found. The general solution of (12), G'/G^2 has three possibilities as enumerated below:

Case-1: (Trigonometric function solutions)

For $\mu\lambda > 0$,

$$\frac{G'}{G^2} = \sqrt{\frac{\mu}{\lambda}}\left[\frac{C\cos\sqrt{\mu\lambda}\,\xi + D\sin\sqrt{\mu\lambda}\,\xi}{D\cos\sqrt{\mu\lambda}\,\xi - C\sin\sqrt{\mu\lambda}\,\xi}\right]. \tag{13}$$

Case-2: (Hyperbolic function solutions)

If we have $\mu\lambda < 0$, then

$$\frac{G'}{G^2} = -\frac{\sqrt{|\mu\lambda|}}{\lambda}\left[\frac{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) + D}{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) - D}\right]. \tag{14}$$

Case-3: (Rational function solutions)

When $\mu = 0$, $\lambda \neq 0$, one recovers

$$\frac{G'}{G^2} = -\frac{C}{\lambda(C\xi + D)}, \tag{15}$$

where C and D are constants. Three types of the solution can be obtained by substituting the values of unknowns a_0, α_n, $\beta_n (n = 1, 2, 3, \ldots, N)$ and the ratios ((13)–(15)) into Eq. (11).

3. Application to perturbed KE equation

In order to obtain exact traveling wave solutions of perturbed KE equation, two integration approaches are utilized. They are $\exp(-\phi(\xi))$–expansion algorithm and G'/G–expansion method. The following forms traveling wave solution is assumed [1–4,6,8,10]:

$$u(x, \tau) = U(\xi)e^{i(-\mathrm{k}\,x+w\tau+\theta_0)}, \quad \xi = x - v\tau, \tag{16}$$

where v is the soliton velocity, k is the soliton frequency, w is the soliton wave number and θ_0 is the phase constant. Inserting (16) into (1) and splitting into real and imaginary parts yields a pair of relations. The imaginary part gives

$$(v + \alpha + 2\mathrm{a\,k}) + (3\lambda_1 + 2\mu_1)U^2 = 0. \tag{17}$$

After setting the coefficients of linearly independent functions to zero we arrive at:

$$v = -\alpha - 2\mathrm{ak}, \tag{18}$$

and the constraint condition

$$3\lambda_1 + 2\mu_1 = 0. \tag{19}$$

Next, the real part gives

$$aU'' - (w + \mathrm{a}k^2 + \alpha k)U + 2cU^2U' - k\lambda_1 U^{2m+1} + bU^5 = 0. \tag{20}$$

For closed–form solution, the following transformation is adopted:

$$U = V^{1/2}$$

that will reduce Eq. (20) into the following ODE

$$a(2VV'' - (V')^2) - 4(w + ak^2 + \alpha k)V^2 + 4cV^2V' - 4\lambda_1 kV^{m+2} + 4bV^4 = 0. \tag{21}$$

This ODE will be now analyzed further along using the two schemes in the subsequent subsections.

3.1. exp $(-\phi(\xi))$-expansion method

Balancing the highest order derivative and nonlinear term in Eq. (21) gives $N = 1$. Then Eq. (4) reduces to

$$V(\xi) = a_0 + a_1 \exp(-\Phi(\xi)), \tag{22}$$

where a_0 and a_1 are constants that are to be determined. Substituting Eq. (22) into Eq. (21) yields a polynomial in $\exp(-\Phi(\xi))$. Equating the coefficients of powers of $\exp(-\Phi(\xi))$ to zero gives a system of relations. The following sets of solutions are attained.

SET 1

$$a_1 = \frac{c \pm \sqrt{-3ab + c^2}}{2b},$$

$$a_0 = \mp\frac{3a(4ab\lambda - c^2\lambda + ck\lambda_1 \mp 2\sqrt{-3ab + c^2}k\lambda_1)}{4(4ab - c^2)(\mp c + \sqrt{-3ab + c^2})},$$

$$w = \mp\frac{k(\pm 4(-4ab + c^2)^2(ak + \alpha) + a(\pm 12ab + c(\mp 5c + 4\sqrt{-3ab + c^2}))k\lambda_1^2)}{4(-4ab + c^2)^2},$$

$$\mu = \frac{(-4ab + c^2)^2\lambda^2 + (12ab \pm c(\mp 5c + 4\sqrt{-3ab + c^2}))k^2\lambda_1^2}{4(-4ab + c^2)^2}.$$

SET 2

$$a_1 = \frac{c \pm \sqrt{-3ab + c^2}}{2b},$$

$$a_0 = 0,$$

$$\mu = 0,$$

$$\lambda = \pm\frac{(\mp c + 2\sqrt{-3ab + c^2}k\lambda_1)}{4ab - c^2},$$

$$w = \mp\frac{k(\pm 4(-4ab + c^2)^2(ak + \alpha) + a(\pm 12ab + c(\mp 5c + 4\sqrt{-3ab + c^2}))k\lambda_1^2)}{4(-4ab + c^2)^2}.$$

The following solutions can be obtained for **SET 1**:

When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, singular optical solitons evolve as:

$$u(x, \tau) = e^{i(-kx + w\tau + \theta_0)}\left[\mp\frac{3a(4ab\lambda - c^2\lambda + ck\lambda_1 \mp 2\sqrt{-3ab + c^2}k\lambda_1)}{4(4ab - c^2)(\mp c + \sqrt{-3ab + c^2})} + \frac{(c \pm \sqrt{-3ab + c^2}\mu)}{b\left[-\lambda - \sqrt{-4\mu + \lambda^2}\tanh\left[\frac{1}{2}\sqrt{-4\mu + \lambda^2}(C + \xi)\right]\right]}\right]. \tag{23}$$

When $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, then periodic singular solutions are:

$$u(x, \tau) = e^{i(-kx + w\tau + \theta_0)}\left[\mp\frac{3a(4ab\lambda - c^2\lambda + ck\lambda_1 \mp 2\sqrt{-3ab + c^2}k\lambda_1)}{4(4ab - c^2)(\mp c + \sqrt{-3ab + c^2})} + \frac{(c \pm \sqrt{-3ab + c^2}\mu)}{b\left[-\lambda - \sqrt{4\mu - \lambda^2}\tan\left[\frac{1}{2}\sqrt{4\mu - \lambda^2}(C + \xi)\right]\right]}\right]. \tag{24}$$

If $\lambda^2 - 4\mu > 0$ and $\mu = 0$ and $\lambda \neq 0$ then bright–singular combo solitons are:

$$u(x, \tau) = e^{i(-kx + w\tau + \theta_0)}\left[\mp\frac{3a(4ab\lambda - c^2\lambda + ck\lambda_1 \mp 2\sqrt{-3ab + c^2}k\lambda_1)}{4(4ab - c^2)(\mp c + \sqrt{-3ab + c^2})} + \frac{c \pm \sqrt{-3ab + c^2}\lambda}{2b(\cosh(\lambda(\xi + C)) + \sinh(\lambda(\xi + C)) - 1)}\right]. \tag{25}$$

However, if $\lambda^2 - 4\mu = 0$ and $\mu \neq 0$ and $\lambda \neq 0$, then

$$u(x, \tau) = e^{i(-kx + w\tau + \theta_0)}\left[\frac{3a((-4ab + c^2)\lambda \mp (\mp c + 2\sqrt{-3ab + c^2})k(-1 + \lambda(C + \xi))\lambda_1)}{4(4ab - c^2)(\mp c + \sqrt{-3ab + c^2})(-1 + \lambda(C + \xi))}\right]. \tag{26}$$

Finally, whenever $\lambda^2 - 4\mu = 0$ and $\mu = 0$ and $\lambda = 0$, then

$$u(x,\tau) = e^{i(-\mathrm{k}\,x + w\tau + \theta_0)}\left[\frac{c \pm \sqrt{-3\mathrm{ab} + c^2}}{2b(C+\xi)} \mp \frac{3a(\mathrm{ck}\lambda_1 \mp 2\sqrt{-3\mathrm{ab} + c^2}\,k\lambda_1)}{4(4\mathrm{ab} - c^2)(\mp c + \sqrt{-3\mathrm{ab} + c^2})}\right], \tag{27}$$

where C is the constant of integration.

The following solutions can be obtained for **SET 2**, as

When $\lambda^2 - 4\mu > 0$ and $\mu = 0$ and $\lambda \neq 0$ then bright–singular combo solitons are listed as:

$$u(x,\tau) = e^{i(-\mathrm{k}\,x + w\tau + \theta_0)}\left[\frac{c \pm \sqrt{-3\mathrm{ab} + c^2}\,\lambda}{2b(-1 + \cosh[\lambda(\xi + C)] + \sinh[\lambda(\xi + C)])}\right]. \tag{28}$$

However, if $\lambda^2 - 4\mu = 0$ and $\mu = 0$ and $\lambda = 0$, then rational solutions are:

$$u(x,\tau) = e^{i(-\mathrm{k}\,x + w\tau + \theta_0)}\left[\frac{c \pm \sqrt{-3\mathrm{ab} + c^2}}{2b(\xi + C)}\right], \tag{29}$$

where C is the constant of integration.

3.2. G'/G^2-expansion method

Balancing the nonlinear terms and highest order derivative terms, in Eq. (21), gives $N = 1$. The solution of Eq. (21), can be written as

$$V(\xi) = a_0 + a_1\left(\frac{G'}{G^2}\right) + b_1\left(\frac{G'}{G^2}\right)^{-1}, \tag{30}$$

where a_0, a_1, b_1 are constants to be determined. Substitute Eq. (30) into Eq. (21) and then collect coefficients of like powers of $\left(\frac{G'}{G^2}\right)^j$, ($j = 0, \pm 1, \pm 2, \pm 3, \pm 4$). A set of nonlinear algebraic equations follows after equating each coefficient to zero. The resulting algebraic system is solved with the help of Mathematica to get the values of unknown constants a_0, a_1, b_1. Following sets of solution are recovered.

SET 1:

$$a_0 = \pm\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab} + c^2)\lambda^2\mu^2})}{b^2}},$$

$$a_1 = -\frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab} + c^2)\lambda^2\mu^2}}{2\mathrm{b}\ c\mu},$$

$$b_1 = \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab} + c^2)\lambda^2\mu^2}}{2\mathrm{b}\ c\lambda}.$$

SET 2

$$a_0 = \pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab} + c^2)\lambda^2\mu^2})}{b^2}},$$

$$a_1 = -\frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab} + c^2)\lambda^2\mu^2}}{2\mathrm{b}\ c\mu},$$

$$b_1 = 0.$$

SET 3

$$a_0 = \pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab} + c^2)\lambda^2\mu^2})}{b^2}},$$

$$a_1 = 0,$$

$$b_1 = \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab} + c^2)\lambda^2\mu^2}}{2\mathrm{b}\ c\lambda}.$$

The following solutions can be obtained corresponding to **SET 1**:

Case-1: (Trigonometric function solutions)

If $\mu\lambda > 0$,

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} - \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}\mu}\left(\sqrt{\frac{\mu}{\lambda}}\left(\frac{C\cos\sqrt{\mu\lambda}\,\xi + D\sin\sqrt{\mu\lambda}\,\xi}{D\cos\sqrt{\mu\lambda}\,\xi - C\sin\sqrt{\mu\lambda}\,\xi}\right)\right) + \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}\lambda}\left(\sqrt{\frac{\mu}{\lambda}}\left(\frac{C\cos\sqrt{\mu\lambda}\,\xi + D\sin\sqrt{\mu\lambda}\,\xi}{D\cos\sqrt{\mu\lambda}\,\xi - C\sin\sqrt{\mu\lambda}\,\xi}\right)\right)^{-1}\Bigg]. \tag{31}$$

To obtain periodic–singular solutions, choose $C = D$, $\mu = 1$, $\lambda = 1$, and this yields:

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\left[-\sqrt{\frac{3\mathrm{ab} - 2(c^2 + \sqrt{c^2(-3\mathrm{ab}+c^2)})}{b^2}} - \frac{(c^2 + \sqrt{c^2(-3\mathrm{ab}+c^2)}\tan[2\xi])}{\mathrm{bc}}\right]. \tag{32}$$

Case-2 (Hyperbolic function solutions):
If $\mu\lambda < 0$,

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} - \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}\mu}\left(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\left(\frac{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) + D}{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) - D}\right)\right) + \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}\lambda}\left(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\left(\frac{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) + D}{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) - D}\right)\right)^{-1}\Bigg]. \tag{33}$$

To obtain soliton solution, choose $C = D$, to yield dark–singular optical solitons as:

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} - \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}}\left(-\frac{\coth[\sqrt{\lambda\mu}\,\xi]}{\sqrt{\lambda\mu}}\right) + \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}}\left(-\frac{\tanh[\sqrt{\lambda\mu}\,\xi]}{\sqrt{\lambda\mu}}\right)\Bigg]. \tag{34}$$

The following solutions corresponding to **SET 2**.
Case-1 (Trigonometric function solutions):
If $\mu\lambda > 0$,

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} - \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}\mu}\left(\sqrt{\frac{\mu}{\lambda}}\left(\frac{C\cos\sqrt{\mu\lambda}\,\xi + D\sin\sqrt{\mu\lambda}\,\xi}{D\cos\sqrt{\mu\lambda}\,\xi - C\sin\sqrt{\mu\lambda}\,\xi}\right)\right)\Bigg]. \tag{35}$$

Case-2 (Hyperbolic function solutions):
If $\mu\lambda < 0$, then

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} - \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}\mu}\left(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\left(\frac{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) + D}{C\sinh(2\sqrt{|\mu\lambda|}\,\xi) + C\cosh(2\sqrt{|\mu\lambda|}\,\xi) - D}\right)\right)\Bigg]. \tag{36}$$

To obtain soliton solution, choose $C = D$, to display singular optical solitons

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu - 2(c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} - \frac{c^2\lambda\mu + \sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b\,c}}\left(-\frac{\coth[\sqrt{\lambda\mu}\,\xi]}{\sqrt{\lambda\mu}}\right)\Bigg]. \tag{37}$$

The following solutions corresponding to **SET 3**.
Case-1 (Trigonometric function solutions):
If $\mu\lambda > 0$,

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu-2(c^2\lambda\mu+\sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} + \frac{c^2\lambda\mu+\sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b}\,c\lambda}\left(\sqrt{\frac{\mu}{\lambda}}\left(\frac{C\cos\sqrt{\mu\lambda}\,\xi+D\sin\sqrt{\mu\lambda}\,\xi}{D\cos\sqrt{\mu\lambda}\,\xi-C\sin\sqrt{\mu\lambda}\,\xi}\right)\right)^{-1}\Bigg]. \tag{38}$$

Case-2 (Hyperbolic function solutions):
If $\mu\lambda < 0$,

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu-2(c^2\lambda\mu+\sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} \frac{c^2\lambda\mu+\sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b}\,c\lambda}\left(-\frac{\sqrt{|\mu\lambda|}}{\lambda}\left(\frac{C\sinh(2\sqrt{|\mu\lambda|}\,\xi)+C\cosh(2\sqrt{|\mu\lambda|}\,\xi)+D}{C\sinh(2\sqrt{|\mu\lambda|}\,\xi)+C\cosh(2\sqrt{|\mu\lambda|}\,\xi)-D}\right)\right)^{-1}\Bigg]. \tag{39}$$

To obtain soliton solution, choose $C = D$, and the following singular optical soliton solution is exhibited:

$$u(x,\tau) = e^{i(-\mathrm{kx}+w\tau+\theta_0)}\Bigg[\pm\frac{1}{2}\sqrt{\frac{3\mathrm{ab}\lambda\mu-2(c^2\lambda\mu+\sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2})}{b^2}} \frac{c^2\lambda\mu+\sqrt{c^2(-3\mathrm{ab}+c^2)\lambda^2\mu^2}}{2\mathrm{b}\,\mathrm{c}}\left(-\frac{\coth[\sqrt{\lambda\mu}\,\xi]}{\sqrt{\lambda\mu}}\right)\Bigg]. \tag{40}$$

4. Conclusions

This paper secured chirp–free dark and singular optical soliton solutions, as well as their combinations, to the perturbed KE equation where the perturbation terms are all of Hamiltonian type. Two integration algorithms are adopted and these schemes revealed such soliton solutions. The constraint conditions on the parameters are also exhibited that display existence criteria to these soliton solutions. It is unfortunate that bright soliton solutions are not recoverable with either of these algorithms.. It is only extended trial function method that enables the retrieval of bright soliton solutions to the model. This was in fact already reported earlier [2]. In future, further work needs to be conducted. The soliton perturbation theory is to be established along with quasi–monochromatic soliton solutions. Later on, Lie symmetry approach will be implemented to secure additional form of solutions to the model that could be applicable to the photonics arena. These will be reported with time.

Conflict of interest

The authors also declare that there is no conflict of interest.

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