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Eigenvalue approach for generalized thermoelastic porous medium under the effect of thermal loading due to a laser pulse in DPL model

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Abstract: The aim of the present study is concerned with the thermal loading due to laser pulse on thermoelastic porous medium under dual-phase-lag model DPL. The material is a homogeneous isotropic elastic half-space and heated by a non-Gaussian laser beam with the pulse duration of 8 ps. Eigenvalue approach is proposed to analyze the problem and obtain the analytical solutions of the displacement components, stresses, temperature distribution and change in the volume fraction field. The results of the physical quantities have been illustrated graphically by the comparison between the coupled theory, DPL model and Lord–Shulman theory of the certain value of time and distance. The results presented in this article should prove useful for researchers in material science (porous media), designers of new materials, physicists as well as for those working on the development of heat laser pulse practical situations as in geophysics, optics, acoustics and geomagnetic.

Keywords: Generalized thermoelastic; Laser pulse; Porous medium; DPL model; Eigenvalue approach

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1. Introduction

To overcome the mathematical discrepancy associated with the infinite speed of propagation of thermoelastic disturbances in the classical coupled thermoelasticity, the improved theories which admit finite speed for thermoelastic signals have been formulated either by incorporating a heat flux rate term into Fourier's law or by including the temperature rate among the constitutive variables. The resulting theories are referred to as the coupled thermoelasticity with second sound effect, or the generalized theory of coupled thermoelasticity CT due to Biot [1]. The basis of the model proposed by Lord and Shulman [2] is to modify the Fourier's law of the heat conduction by introducing the concept of a relaxation time. They predict the finite velocity of propagation to heat field. The second generalization was developed by Green and Lindsay [3]. This theory contains two constants that act as relaxation times and modifies all the equations of coupled theory, not the heat conduction equation only.

The dual-phase-lag DPL model, which describes the interactions between photons and electrons on the microscopic level as regards sources causing a delayed response on the macroscopic scale, was established by Tzou [4]. For the macroscopic formulation, it would be convenient to use the DPL model for the investigation of micro-structural effect on the behavior of heat transfer. The thermoelastic interactions in a semi-infinite medium subjected to a Ramp-type heating with the DPL model was studied by Abbas and Zenkour [5]. The DPL model to study the rotation effect on a linear micro-polar thermoelastic isotropic medium with two temperatures was used by Othman et al. [6]. Laser at a high intensity when interacts with the solid surface, the absorption takes place. This in turn causes an internal energy gain of the substrate material and heat release from the irradiated region. Since the process, in general, is fast, the temperature gradients remain high in

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the irradiated region. This results in high thermal strain and thermally induced stresses in this region. The exact solutions for 2D problem of an anisotropic infinite thermoelastic plate heated laser pulse were presented by Al-Qahtani and Datta [7]. The reflection of plane waves from electro-magneto-thermoelastic half-space with the DPL model was studied by Abd-Alla et al. [8]. The effect of heat laser pulse on generalized thermoelasticity of micro-polar medium was investigated by Othman and Marin [9].

The distributed body concept introduced by Goodmann and Cowin [10], in the context of the granular and porous materials asserts that the mass density has the decomposition $l\phi$ where l is the density of the matrix material and ϕ is the volume fraction filed. This representation introduces an additional degree of kinematic freedom. A nonlinear theory to describe the properties of homogeneous elastic materials with voids free of fluid was established by Nunziato and Cowin [11]. Moreover, the theory of Cowin and Nunziato was a more appropriated theory than other theories for the study of special continuum and geological materials, such as rocks, soils, and manufactured porous materials like ceramics and pressed powders. Generally, this theory based on the balance of energy, where the presence of the pores or voids involves an additional degree of freedom, was called the fraction of elementary volume. A theory to describe the linear elastic materials with voids was investigated by Cowin and Nunziato [12]. The linear theory of thermoelastic materials with voids was developed by Iesan [13]. The response of thermal source in initially stressed generalized thermoelastic half-space with voids was studied by Abbas and Kumar [14]. The effect of gravity on generalized thermoelastic diffusion due to laser pulse using dual-phase-lag model was discussed by Othman and Eraki [15]. The finite element method and other methods in different generalized thermoelastic problems have been applied by many authors (see for instant Ref. [16–19]). Several problems of porous medium have studied as [20–30].

The present investigation is to determine the components of the displacement, the stress, the temperature distribution and the change in the volume fraction field in an isotropic homogeneous thermoelastic porous medium subjected to thermal loading due to laser pulse under DPL model. The results have been performed and presented graphically with concluding remarks.

2. Formulation of the problem and basic equations

We consider a homogeneous, isotropic, thermoelastic porous material with a half-space $(y \geq 0)$ as in Fig. 1. The rectangular Cartesian coordinate system (x, y, z) has originated on the surface $(y = 0)$. For two-dimensional

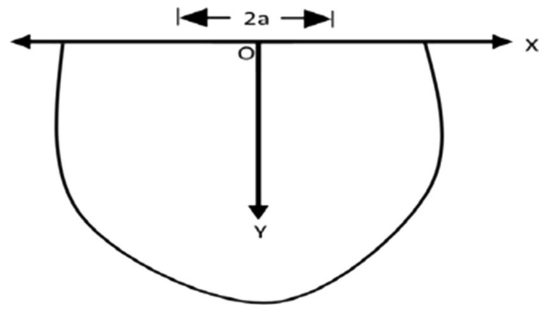

Fig. 1 Geometry and coordinate system

problem, we assume the dynamic displacement vector as $\boldsymbol{u} = (u, v, 0)$. All quantities considered will be a function of the time variable t and of the coordinates x and y.

Following Cowin and Nunziato [12], the field equations and constitutive relations for linear homogenous, isotropic generalized thermoelastic solid with voids without body forces, heat sources and extrinsic equilibrated body force can be written in the context of DPL model, as [17]

$$\mu\,\nabla^2\boldsymbol{u} + (\lambda + \mu)\,\nabla(\nabla.\,\boldsymbol{u}) + b\,\nabla\phi - \beta\,\nabla T = \rho\ddot{\boldsymbol{u}}, \tag{1}$$

$$\alpha\,\nabla^2\phi - b\,\nabla\cdot\boldsymbol{u} - \xi_1\,\phi - \omega_0\,\dot{\phi} + m\,T = \rho\,\psi\,\ddot{\phi}, \tag{2}$$

$$K\left(1 + \tau_\theta\frac{\partial}{\partial t}\right)\nabla^2 T = \left(1 + \tau_q\frac{\partial}{\partial t}\right)(\rho\,C_E\dot{T} + mT_0\phi + T_0\beta\,\nabla\cdot\dot{\boldsymbol{u}} - Q), \tag{3}$$

$$\sigma_{ij} = \lambda\,\nabla\cdot\boldsymbol{u}\,\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + b\phi\delta_{ij} - \beta T\delta_{ij}. \tag{4}$$

where $e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$, are the components of strain tensor, $(i, j = 1, 2, 3)$, λ, μ are the Lame' constants, $\alpha,\ b,\ \xi_1,\ \omega_0,\ m,\ \psi$ are the material constants due to presenceof voids, T is the temperature distribution, $\boldsymbol{u}$ is the displacement vector, $\beta = (3\lambda + 2\mu)\alpha_t$ such that α_t is the coefficient of thermal expansion, ρ is the density, C_E is the specific heat, k is the thermal conductivity, T_0 is the reference temperature chosen so that $|(T - T_0)/T_0| << 1$, ϕ is the change in the volume fraction field, σ_{ij} are the components of the stress tensor, δ_{ij} is the Kronecker delta, t is the time variable, a dot denotes differentiation with respect to time, a comma denotes material derivatives, and Q is the heat input of the laser pulse. The plate surface is influenced by laser pulse given by the heat input [9]

$$Q(x, y, t) = \frac{I_0\gamma^*\,t}{2\pi r^2 t_0^2}\exp\left(-\frac{x^2}{r^2} - \frac{t}{t_0}\right)\exp(-\gamma^* y). \tag{5}$$

where the energy absorbed is I_0, t_0 is the pulse rise time, r is the beam radius, and γ^* is the absorption depth of heating energy.

The components of stress tensor can be expressed by: