Fundamentals of Electrical Engineering

For first year Civil Engineering Dep.,

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Lecture 5
METHODS OF CIRCUITS ANALYSIS

Thevenin's and Norton Theorem, Maximum Power

THEVENIN'S THEOREM

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

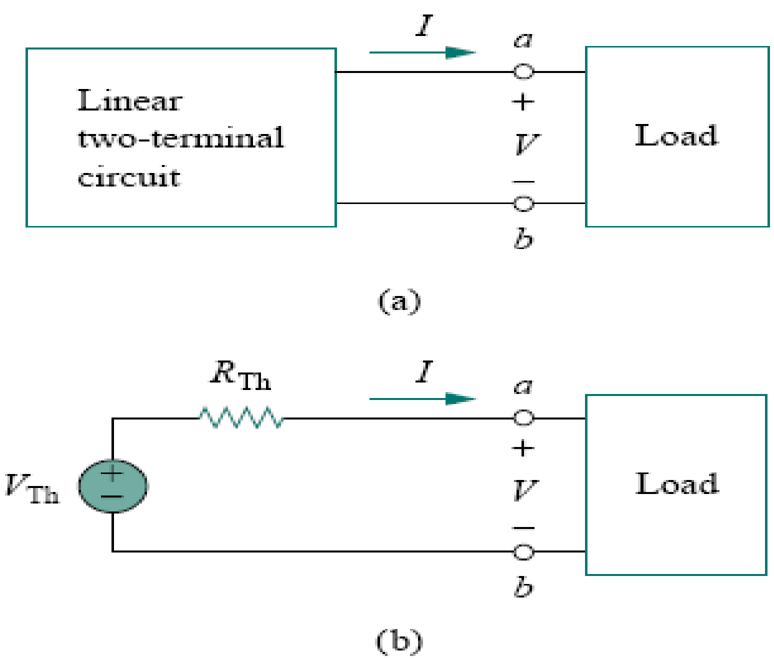

Figure 4.23 Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

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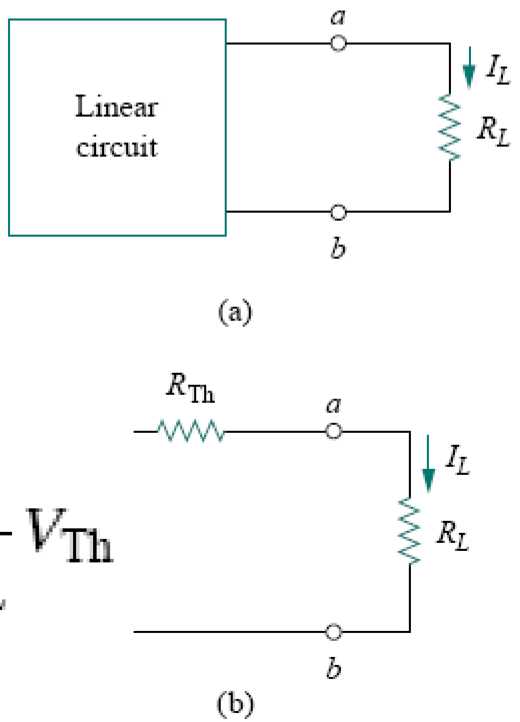


$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

Figure 4.26 A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

EXAMPLE 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16,$ and 36 Ω.

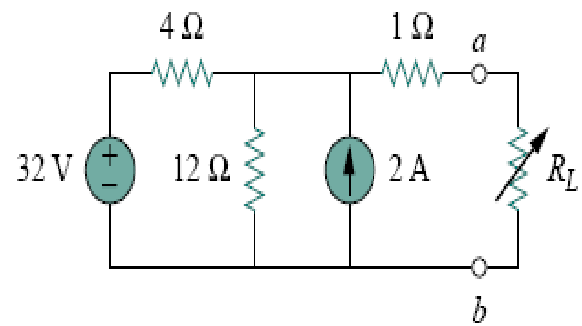


Figure 4.27 For Example 4.8.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

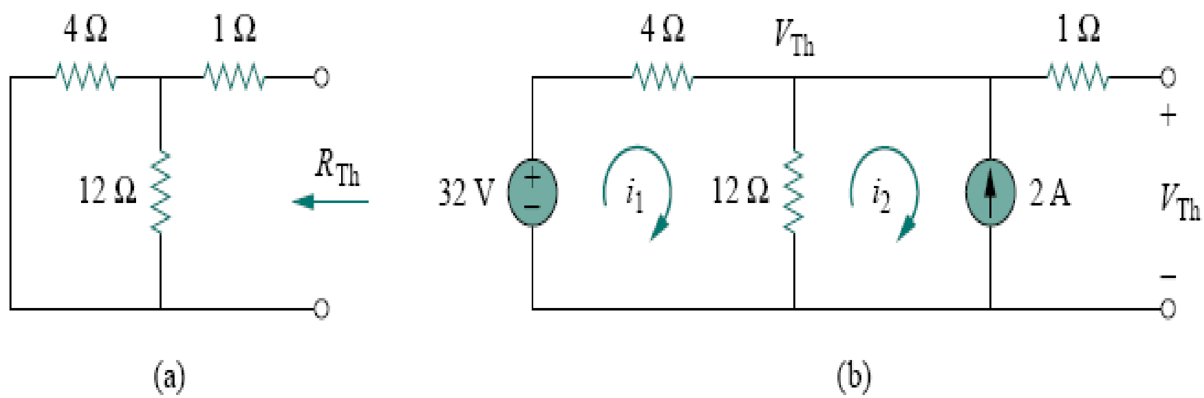
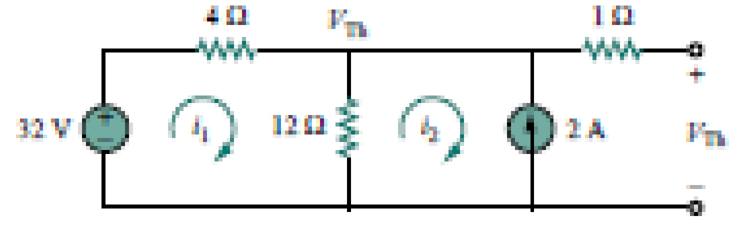


Figure 4.28 For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.



(b)

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

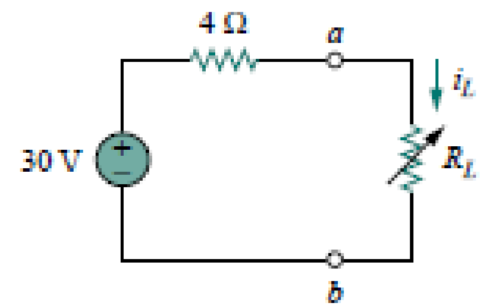
$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \qquad \Longrightarrow \qquad V_{Th} = 30\text{ V}$$

as obtained before. We could also use source transformation to find V_{Th}.

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is



$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3\text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{20} = 1.5\text{ A}$$

$$I_L = \frac{30}{40} = 0.75\text{ A}$$

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

PRACTICE PROBLEM 4.10

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Answer: $V_{Th} = 0$ V, $R_{Th} = -7.5\ \Omega$.

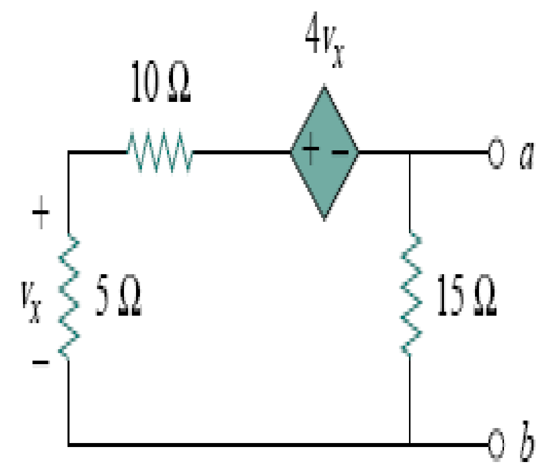


Figure 4.36 For Practice Prob. 4.10.

NORTON'S THEOREM

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

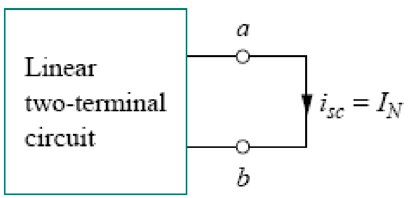


Figure 4.38 Finding Norton current I_N.

$$R_N = R_{Th}$$

$$I_N = i_{sc}$$

$$I_N = \frac{V_{Th}}{R_{Th}}$$

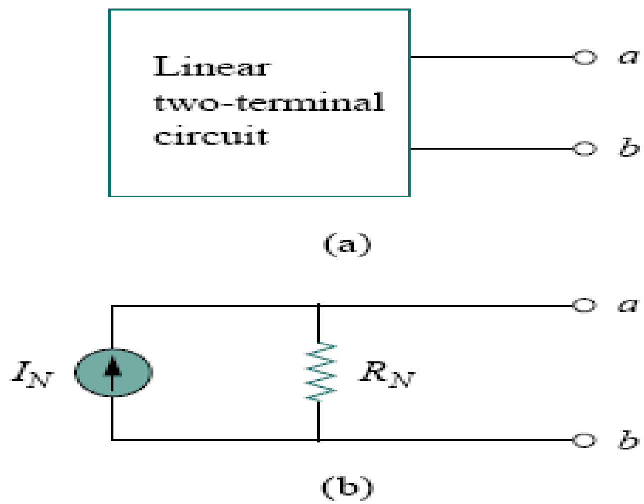


Figure 4.37 (a) Original circuit, (b) Norton equivalent circuit.

EXAMPLE 4.11

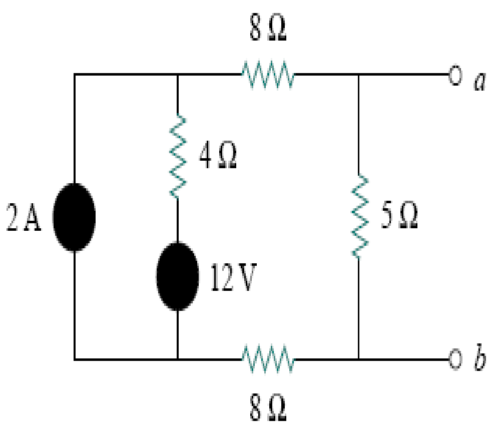


Figure 4.39 For Example 4.11.

Find the Norton equivalent circuit of the circuit in Fig. 4.39.

Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N. Thus,

$$R_N = 5 \parallel (8+4+8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals a and b, as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Longrightarrow \quad i_4 = 0.8 \text{ A}$$

$$v_{oc} = V_{\text{Th}} = 5i_4 = 4 \text{ V}$$

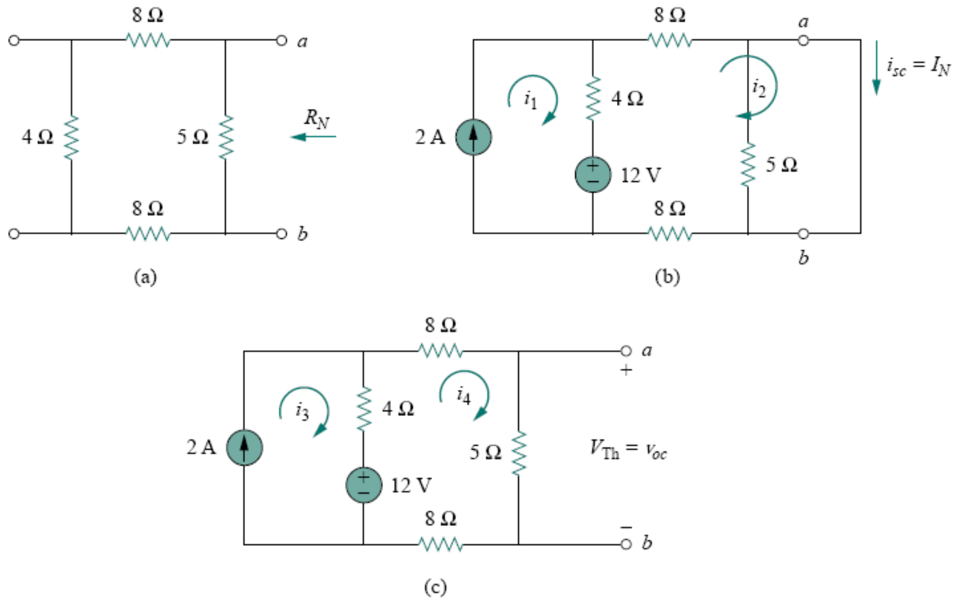


Figure 4.40 For Example 4.11; finding: (a) R_N, (b) $I_N = i_{sc}$, (c) $V_{\text{Th}} = v_{oc}$.

Hence,

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.7) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

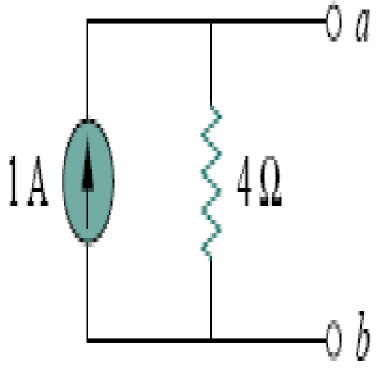


Figure 4.41 Norton equivalent of the circuit in Fig. 4.39.

PRACTICE PROBLEM 4.11

Find the Norton equivalent circuit for the circuit in Fig. 4.42.

Answer: $R_N = 3\ \Omega$, $I_N = 4.5$ A.

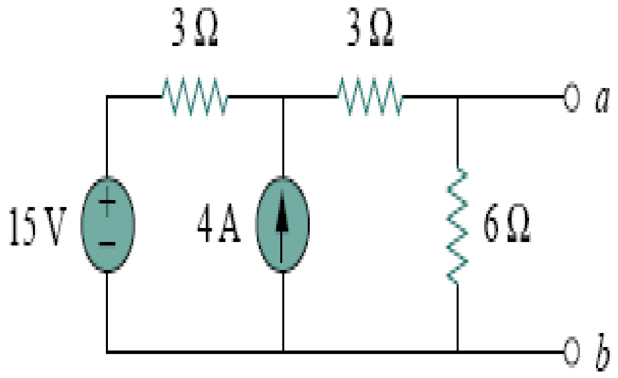
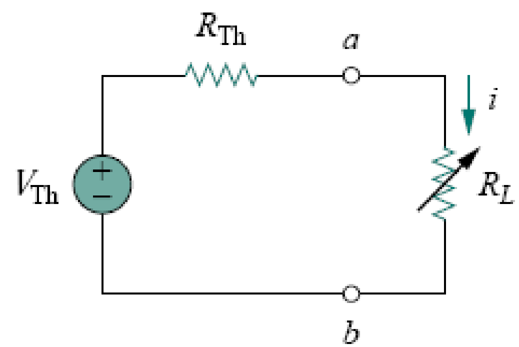


Figure 4.42 For Practice Prob. 4.11.

MAXIMUM POWER TRANSFER

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$



48 The circuit used for m power transfer.

$$R_L = R_{Th}$$

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

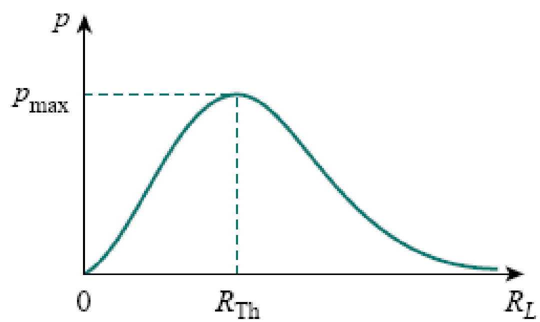


Figure 4.49 Power delivered to the load as a function of R_L.

EXAMPLE

Find the value of *RL for maximum power transfer in the circuit of Fig.* 4.50. Find the maximum power.

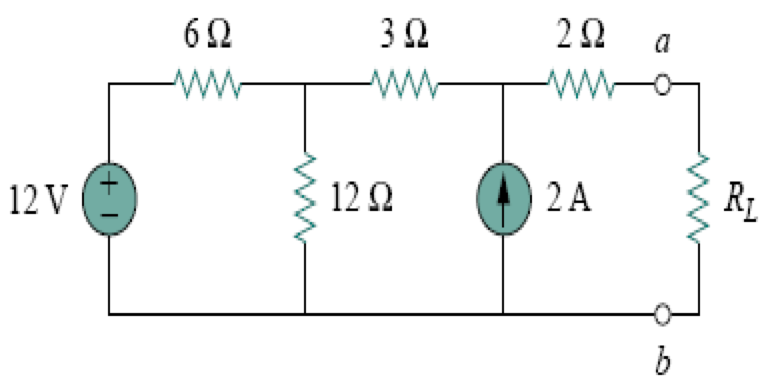


Figure 4.50 For Example 4.13.

Solution:

We need to find the Thevenin resistance *RTh and the Thevenin voltage*

VTh across the terminals a-b. To get RTh, we use the circuit in Fig. 4.51(a)

and obtain

$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

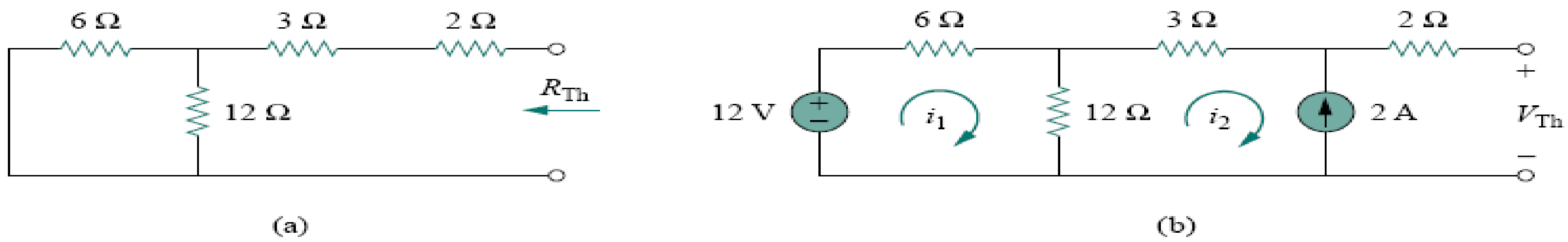


Figure 4.51 For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2 \text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \qquad \Longrightarrow \qquad V_{Th} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

Thank you

With my best regards