Fractional Order Generalized Thermoelasticity in an Unbounded Medium with Cylindrical Cavity

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Abstract: This paper is concerned with the investigation of the temperature, displacement, and stresses due to thermal shock loading on the inner surface cavity in an infinite medium with a cylindrical cavity. The governing equations will be taken into the context of the fractional order generalized thermoelasticity theory. In the Laplace transform domain, the form of a vector-matrix differential equation has been written for the basic equations, which is then solved by an eigenvalue approach. The result provides a motivation to investigate the effect of fractional parameter on the physical quantities. **DOI: 10.1061/(ASCE)EM.1943-7889.0001071.** *© 2016 American Society of Civil Engineers.*

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Introduction

The theory of generalized thermoelasticity has drawn the attention of researchers due to its applications in various diverse fields such as nuclear reactor's design, earthquake engineering, high-energy particle accelerators. The first of such modeling is the extended thermoelasticity (LS) theory of Lord and Shulman (1967), who established the generalization of thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier law. Green and Lindsay (1972) proposed the temperature-rate-dependent thermoelasticity (GL) theory with two relaxation times. In the last decade, on-isothermal problems of the theory of elasticity have become increasingly important. First, in the nuclear field, the extremely high temperature and temperature gradients originating inside nuclear reactors influence their design and operations.

Second, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. The counterparts of this problem in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods (Dhaliwal and Sherief 1980; Sherief and Anwar 1988, 1989; Sherief et al. 2004; Lee 2006; Abbas 2012, 2014b, c, d, a; Abbas and Othman 2012; Hosseini and Abolbashari 2012; Othman and Abbas 2012; Islam et al. 2013; Sherief and El-Maghraby 2013; Xie et al. 2013; Abbas and Zenkour 2014).

The fractional calculus and the fractional differential equations serve as mathematical objects describing many real-world systems. Various approaches and definitions of fractional derivatives have become the main focus of many studies. Youssef (2010) and Youssef and Al-Lehaibi (2010) introduced the fractional order generalized thermoelasticity of both weak and strong heat conductivity, and the corresponding variational theorem for fractional order generalized thermoelasticity was developed. The theory was subsequently employed to solve two-dimensional thermal shock problems using Laplace and Fourier transforms (Youssef 2012). Ezzat (2011a) and Ezzat and El-Karamany (2011c, d) used a Taylor expansion of time-fractional order to introduced a new model of fractional heat equation. Sherief et al. (2010) introduced another model, which is based on the law of heat conduction. Sherief and Abd El-Latief (2013) investigated the effect of variable thermal conductivity on a half-space under the fractional order theory of thermoelasticity. Kumar et al. (2013) studied the fractional order derivative on plane deformation in a thermoelastic media due to thermal source.

The idea of the present work is to investigate the thermal shock problem of fractional order generalized thermoelasticity of an unbounded medium with cylindrical cavity. The nondimensional equations are handled by employing an analytical-numerical technique based on the Laplace transform and an eigenvalues approach. Numerical results for the temperature distribution, displacement, radial stress, and hoop stress are represented graphically.

Basic Equation and Formulation of the Problem

Following the literature (El-Karamany and Ezzat 2011; Ezzat 2011b), the basic equations of generalized thermoelasticity theory with fractional order for homogeneous isotropic material in the absence of heat sources and body forces are considered as the equation of motion

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

and the equation of heat conduction

$$(K_{ij}T_{,j})_{,i} = \left[\frac{\partial}{\partial t} + \frac{\tau_o^\alpha}{\Gamma(\alpha+1)}\frac{\partial^{1+\alpha}}{\partial t^{1+\alpha}}\right](\rho c_e T + \gamma T_0 e), \qquad 0 < \alpha \le 1 \tag{2}$$

where

$$\frac{\partial^\alpha}{\partial t^\alpha} f(r,t) = \begin{cases} f(r,t) - f(r,0) & \alpha \to 0 \\ I^{\alpha-1}\frac{\partial f(r,t)}{\partial t} & 0 < \alpha < 1 \\ \frac{\partial f(r,t)}{\partial t} & \alpha = 1 \end{cases}$$

The constitutive equations are given by

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma(T - T_0)]\delta_{ij} \tag{3}$$

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