Commun Nonlinear Sci Numer Simulat 14 (2009) 1385–1395

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Finite element analysis of hydromagnetic flow and heat transfer of a heat generation fluid over a surface embedded in a non-Darcian porous medium in the presence of chemical reaction

R.A. Mohamed [a,*], Ibrahim A. Abbas [b], S.M. Abo-Dahab [a]

[a] Mathematics Department, Faculty of Science, Qena 83523, Egypt
[b] Mathematics Department, Faculty of Science, Sohag, Egypt

ARTICLE INFO

Article history:
Received 21 October 2007
Received in revised form 4 March 2008
Accepted 3 April 2008
Available online 15 April 2008

PACS:
44.05. +e
44.30. +v

Keywords:
MHD
Non-Darcian porous medium
Heat transfer
Finite element method
Heat generation and chemical reaction

ABSTRACT

An analysis is carried out to study the flow, chemical reaction and mass transfer of a steady laminar boundary layer of an electrically conducting and heat generating fluid driven by a continuously moving porous surface embedded in a non-Darcian porous medium in the presence of a transfer magnetic field. The governing partial differential equations are converted into ordinary differential equations by similarity transformation and are solved numerically by using the finite element method. The results obtained are presented graphically for velocity, temperature and concentration profiles, as well as the Sherwood number for various parameters entering into the problem.

© 2008 Published by Elsevier B.V.

1. Introduction

Boundary-layer flow on a moving continuous surface is an important type of flow occurring in a number of engineering processes. In particular, processes involving the mass transfer effect have been recognized as important principally in chemical processing equipment. The heat and mass transfer problem with or without magnetic field, suction or injection, has been analyzed by several characters. General boundary-layer equations for continuous surfaces have been developed by Sakiadis [1]. Flow in the boundary on a continuous semi-infinite shear moving continuous through an otherwise quiescent environment was first studied theoretically by Sakiadis [2] and experimentally verified. Tsou et al. [3] considered the effect of heat transfer on a continuous moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis. Crane [4] considered a moving strip the velocity of which is proportional to the distance from the slit. Erickson et al. [5] considered the study of heat and mass transfer in the laminar boundary-layer flow of moving flat surface with constant surface velocity and temperature considering the effect of suction/injection. Chen and Stroble [6] considered the effect of a buoyancy-induced pressure gradient in a laminar boundary layer of moving flat surface with constant velocity and temperature. The study was further extended by number of researchers to include other physical features such as suction/injection, magnetic field, heat transfer and combined heat and mass transfer analysis, etc. (see for instance [7–14]).

* Corresponding author.
E-mail addresses: rabdalla_1953@yahoo.com (R.A. Mohamed), ibrahim.abbas@sci.sohag.edu.eg, ibrabbas7@yahoo.com (I.A. Abbas), sdahb@yahoo.com (S.M. Abo-Dahab).

1007-5704/$ - see front matter © 2008 Published by Elsevier B.V.
doi:10.1016/j.cnsns.2008.04.006

A continuous moving surface through a porous medium has many applications such as geothermal reservoirs and petroleum industries (petroleum drillings). The possible use of porous media adjacent to surfaces to enhance heat transfer characteristics have lead to extensive research in heat transfer and flows over bodies embedded in porous media. Masterly works on flow and heat transfer in porous media adjacent to surfaces have used Darcy's law which neglects both boundary and inertia effects which become significant in the presence of a solid boundary and when the flow in the porous medium is considered fast. Vafai and Tien [15] and Hong et al. [16] discussed the importance of these effects in flows over surfaces embedded in porous media.

In the area of the steady flow of a uniform stream of an incompressible viscous fluid over an infinite porous plate subject to suction or blowing various aspects of the problem have been investigated by many authors. Suction and blowing on convective heat transfer over a vertical permeable surface embedded in a porous medium was analyzed by Cheng [17]. In that work an application to warm water discharge along the well or fissure to an aquifer of infinite extent is discussed. Later, Lesnic et al. [18] studied the same problem as in Cheng [17] with Newtonian heating and Postelnica et al. [19] extended a previous work without suction/blowing to the case of permeable surface. The problem is reduced to a single third-order nonlinear ordinary differential equation so-called Cheng and Minkowycz's [20].

One of the limitations of all papers is the neglecting of diffusion equation. However, in many real boundary-layers flow, the flow, the heat transfer and the mass diffusion are always coupled. Besides, in industrial processes involving heat and mass transfer over a moving surface the diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid (see [21–25]). This generation or absorption of species can effect the flow and accordingly the properties and quality of the final product. This fact motivates the present study to provide an investigation taking into account the diffusion equation with a chemical reaction source or sink term. Hady et al. [26] studied the MHD boundary-layer flow and heat transfer over a continuously moving wavy surface in an electrically conducting fluid at rest embedded in a fluid-saturated porous medium.

Recently, Abbas [27] investigated finite element analysis of transient free convection flow over vertical plate.

In the present study, we consider steady two dimensional, hydromagnetic flow, heat and mass transfer of a viscous fluid over a continuously moving porous surface embedded in a non-Darcian porous medium with heat generation effects. The chemical reaction source or sink term is retained in the concentration equation. The applied magnetic field is assumed to be non-uniform and magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. In addition, it is assumed that there is no external electric field and that the electric field due to polarization of charges is negligible.

2. Formulation of the problem

Consider steady, laminar, incompressible two dimensional, boundary-layer flow of an electrically conducting and heat generation fluid over a semi-infinite porous flat surface embedded in a non-Darcian porous medium in the presence of a transverse magnetic field and chemical reaction. Let the x-axis representing the axial or tangential distance be placed along the horizontal plate and the y-axis representing the normal distance be perpendicular to it. Let the plate be moving with constant speed U. The surface of the plate is maintained at a uniform constant temperature T_w and a uniform constant concentration C_w. For above the plate, the fluid is stationary and is kept at a temperature T_∞ and concentration C_∞. The porous is assumed to be uniform (that is, it has a constant porosity and permeability). All thermophysical properties are assumed constant. The magnetic Reynolds numbers assumed small so that the induced magnetic field is neglected. In addition, the external electric field, electric field due to polarization of charges, the Hall effect of magnetohydrodynamics, and viscous and magnetic dissipations are all assumed negligible. The governing boundary equations which are based on the balance laws of mass, linear momentum, energy and concentration for this investigation can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}u - \frac{\nu}{k}u - C_1 u^2, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_e}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty), \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \alpha\frac{\partial^2 C}{\partial y^2} - R(C - C_\infty)^\gamma, \tag{4}$$

where x and y are the dimensional distances along and perpendicular to the plate, respectively, u and v are the components of dimensional velocities along x- and y-directions, respectively, ρ is the fluid density, ν is the kinematics viscosity, k, C_1 and k_e are the permeability of the porous medium, Forcheimer inertia coefficient and effective thermal conductivity, respectively, σ is the electrical conductivity of the fluid, $B(x)$ is the magnetic induction, α, R and γ are the diffusion coefficient, reaction rate constant and reaction order, respectively, T and C are the temperature and concentration of the species of the fluid, respectively and Q is the heat generation/absorption coefficient.

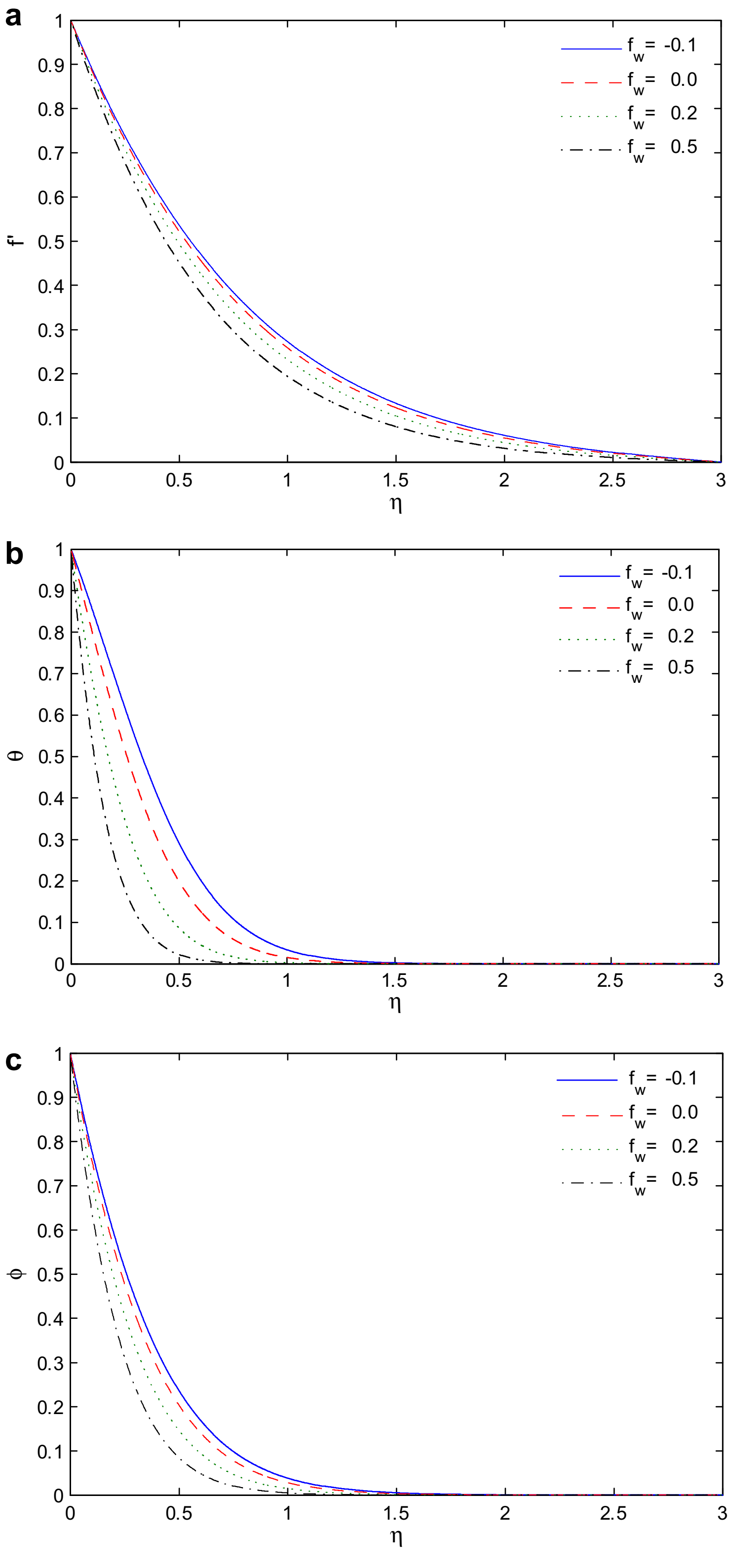

Fig. 1. Effects of f_w on fluid (velocity, temperature and concentration) profiles respect to η with $Sc = 0.5$, $Pr = 10$, $M = 1$, $Da^{-1} = 0.1$, $\gamma = 1$, $\Gamma_x = 0.1$, $\xi = 1$ and $\delta_x = 1$.