It is not difficult to show that, if $\mu_1 = \mu_2$,

$$\eta_2 \cos\theta_i - \eta_1 \cos\theta_t \neq 0 \qquad \text{for any } \theta_i$$

Hence, a perpendicularly polarized incident wave suffers either partial or total reflection.

14.10 PARALLEL POLARIZATION

For *parallel* polarization the electric field vector **E** lies entirely within the plane of incidence, the xz plane as shown in Fig. 14-7. (Thus **E** assumes the role played by **H** in perpendicular polarization.) At the interface,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}$$

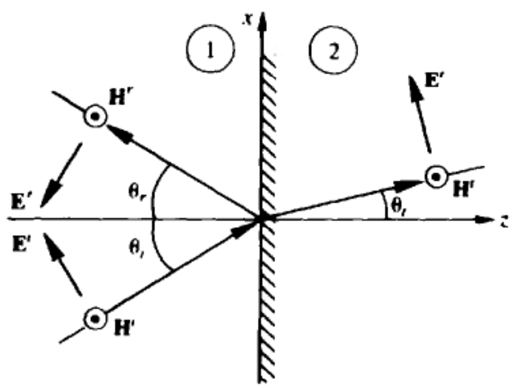

Fig. 14-7

In contrast to perpendicular polarizations, if $\mu_1 = \mu_2$ there will be a particular angle of incidence for which there is no reflected wave. This *Brewster angle* is given by

$$\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

EXAMPLE 6. The Brewster angle for a parallel-polarized wave traveling from air into glass for which $\epsilon_r = 5.0$ is

$$\theta_B = \tan^{-1}\sqrt{5.0} = 65.91°$$

14.11 STANDING WAVES

When waves traveling in a perfect dielectric ($\sigma_1 = \alpha_1 = 0$) are normally incident on the interface with a perfect conductor ($\sigma_2 = \infty$, $\eta_2 = 0$), the reflected wave in combination with the incident wave produces a *standing wave*. In such a wave, which is readily demonstrated on a clamped taut string, the oscillations at all points of a half-wavelength interval are in time phase. The combination of incident and reflected waves may be written

$$\mathbf{E}(z, t) = [E_0^i e^{j(\omega t - \beta z)} + E_0^r e^{j(\omega t + \beta z)}]\mathbf{a}_x = e^{j\omega t}(E_0^i e^{-j\beta z} + E_0^r e^{j\beta z})\mathbf{a}_x$$

Since $\eta_2 = 0$, $E_0^r/E_0^i = -1$ and

$$\mathbf{E}(z, t) = e^{j\omega t}(E_0^i e^{-j\beta z} - E_0^i e^{j\beta z})\mathbf{a}_x = -2jE_0^i \sin\beta z e^{j\omega t}\mathbf{a}_x$$

or, taking the real part,

$$\mathbf{E}(z, t) = 2E_0^i \sin \beta z \sin \omega t \mathbf{a}_x$$

The standing wave is shown in Fig. 14-8 at time intervals of $T/8$, where $T = 2\pi/\omega$ is the period. At $t = 0$, $\mathbf{E} = \mathbf{0}$ everywhere; at $t = 1(T/8)$, the endpoints of the $\mathbf{E}$ vectors lie on sine curve *1*; at $t = 2(T/8)$, they lie on sine curve *2*; and so forth. Sine curves *2* and *6* form an envelope for the oscillations; the amplitude of this envelope is twice the amplitude of the incident wave. Note that adjacent half-wavelength segments are 180° out of phase with each other.

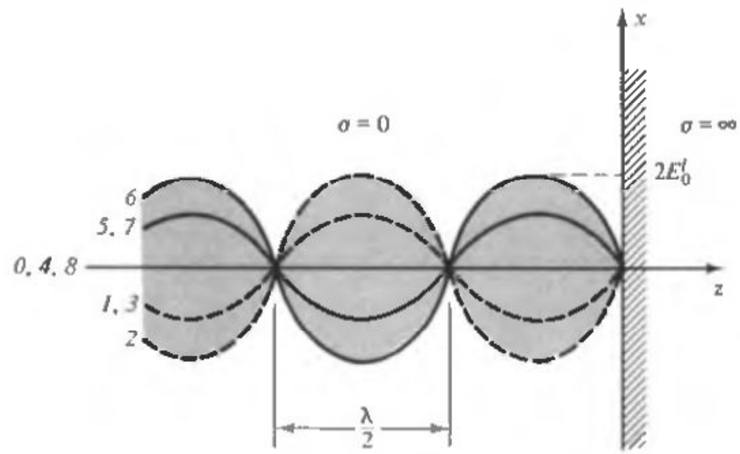

Fig. 14-8

14.12 POWER AND THE POYNTING VECTOR

Maxwell's first equation for a region with conductivity σ is written and then $\mathbf{E}$ is dotted with each term.

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}) = \sigma E^2 + \mathbf{E} \cdot \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

where, as usual, $E^2 = \mathbf{E} \cdot \mathbf{E}$. The vector identity $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$ is employed to change the left side of the equation.

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) - \nabla \cdot (\mathbf{E} \times \mathbf{H}) = \sigma E^2 + \mathbf{E} \cdot \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

By Maxwell's second equation,

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) = \mathbf{H} \cdot \left(-\mu \frac{\partial \mathbf{H}}{\partial t}\right) = -\frac{\mu}{2} \frac{\partial H^2}{\partial t}$$

Similarly,

$$\mathbf{E} \cdot \epsilon \frac{\partial \mathbf{E}}{\partial t} = \frac{\epsilon}{2} \frac{\partial E^2}{\partial t}$$

Substituting, and rearranging terms,

$$\sigma E^2 = -\frac{\epsilon}{2} \frac{\partial E^2}{\partial t} - \frac{\mu}{2} \frac{\partial H^2}{\partial t} - \nabla \cdot (\mathbf{E} \times \mathbf{H})$$